1/D Expansion of Quantum Gravity

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(based on work in preparation with D. Litim)

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Motivations for 1/D

1. The large-N limit, of $SU(N)$ where N is the group degrees of freedom.

2. The analogue of N in quantum gravity is D, space-time dimensions where they are related such that, $N = D(D-3)/2$. In gravity D also comes from loop integrations.

3. Mass dimensions of gravitational constant is, $[G_N] = 2 - D$ Hence, 4 dimensions is not special. Gravity is perturbatively non-renormalisable for all D>2.

4. Phenomenology, large extra-dim. theories, e.g. Kaluza-Klein models, ADD.

5. Consistent previous work in literature.

1/D Expansion in perturbative QG

A. Strominger, DOI: 10.1103/PhysRevD.24.3082

The basic idea is to make an expansion of any Green's function as;

$$G = \sum_{m,n} G_{mn} \kappa^m \left(\frac{1}{D}\right)^n \qquad \kappa = \frac{1}{8\pi G_N}$$

D: number of dimensions

→ Feynman diagram sum in powers of 1/D.

Result: The graphs that survive the large-D limit are the tree graphs and tree graphs with arbitrary number of bubbles such that no two bubbles touch each other.

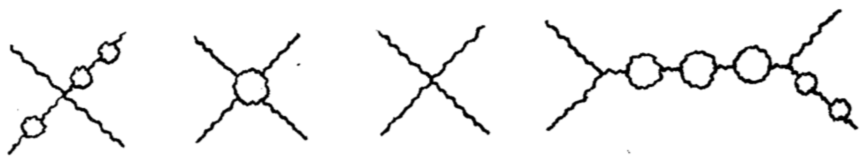


FIG. 3. Some examples of leading contributions to the four-point function. With a rescaled coupling constant, each of these graphs is proportional to $1/D^2$.

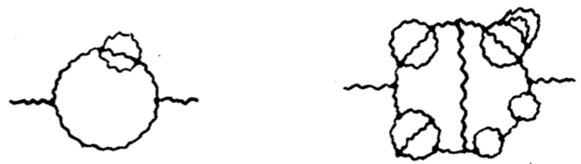


FIG. 4. Nested graphs such as these have very small D-dependent phase-space factors. For large D, they can be neglected relative to disjoint bubble graphs of the same order.

1/D Expansion in Effective Field Theory

N. E. J. Bjerrum-Bohr, arXiv:hep-th/0310263v2

$$\mathcal{L}_{\text{effective EH}} = \int \mathrm{d}^D x \sqrt{-g} \left(\left(\frac{2R}{\kappa^2} + c_1 R^2 + c_2 R^{\mu\nu} R_{\mu\nu} + \cdots \right) + \mathcal{L}_{\text{eff. matter}} \right)$$

In the large-D limit, a particular subset of planar diagrams will carry all leading 1/D contributions to the Green's functions.

Agrees with Strominger's work, except more graphs contributes.

Still:

- Quantum gravity simplifies in a large number of dimensions in effective theory.

1/D Expansion and Lattice Quantum Gravity

H.W. Hamber, R.M.Williams arXiv:hep-th/0512003

- Examination of a 1/D expansion in Lattice QG based on Regge's simplicial construction.
- Scaling exponent ν approaches 0 in the large-D.
- It is concluded that "The action simplifies considerably in the large-D limit."

1/D Expansion in Classical GR

R. Emparan et. al. arXiv:1302.6382

- Interaction between the blackholes reduces with large-D, so the theory becomes non-interacting in the D goes to infinity limit.

- Large-D limit simplifies the theory even in the classical limit of general relativity

Now, we will perform the 1/D expansion with exact renormalisation group techniques.

Renormalisation Group

We start with the Einstein-Hilbert action and we find the scale dependent average effective action and the flow as,

$$\Gamma_k = -\frac{1}{16\pi G_k}\int \mathrm{d}^d x\sqrt{g}\,[R(g) - 2\Lambda_k] + S_{k,\mathrm{gauge}} + S_{k,\mathrm{ghost}}$$

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-1}\partial_t R_k\right] \quad \text{where,} \quad t = \ln k$$

R_k is the IR cutoff, “Tr” represents momentum integration.

And the scale dependent coupling constants are defined as:

$$G_k = G_N Z_k^{-1}$$

$$g = G_k k^{D-2}$$

$$\lambda = k^{-2}\Lambda_k$$

1/D Expansion and Asymptotic Safety

β-functions have been calculated as;

$$\beta_\lambda = (-2+\eta)\lambda + g(a_1 - \eta a_2)$$
$$\beta_g = (D-2+\eta)g$$
$$\eta = \frac{gb_1}{1+gb_2}$$

(*M. Reuter, O. Lauscher arXiv:hep-th/0108040*)

- We rescaled g as g/c_D, where $c_D \equiv (4\pi)^{\frac{D}{2}-1}\Gamma(\frac{D}{2}+2)$
- We want an expansion around 1/D.
- D dependence is coming from the trace multiplications.
- Background Field Technique

(*D.Litim arXiv:hep-th/0312114)*

By using the optimised cutoff function we get the coefficient functions in terms of λ,D and α-the gauge fixing constant- as;

$$a_1(\lambda) = \frac{D(D-1)(D+2)}{2(1-2\lambda)} + \frac{D(D+2)}{1-2\alpha\lambda} - 2D(D+2) \quad (1)$$

$$a_2(\lambda) = \frac{D(D-1)}{2(1-2\lambda)} + \frac{D}{1-2\alpha\lambda} \quad (2)$$

$$\begin{aligned} b_1(\lambda) = & -\frac{1}{3}(1+\frac{2}{D})(D^3+6D+12) + \frac{D(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\ & - \frac{(D+2)(D^3-4D^2+7D-8)}{(D-1)(1-2\lambda)^2} + \frac{(D+2)(D^2-6)}{6(1-2\alpha\lambda)} \\ & - \frac{2(D+2)(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} \end{aligned} \quad (3)$$

$$\begin{aligned} b_2(\lambda) = & \frac{(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} - \frac{D^3-4D^2+7D-8}{(D-1)(1-2\lambda)^2} \\ & + \frac{(D+2)(D^2-6)}{6D(1-2\alpha\lambda)} - \frac{2(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} \end{aligned} \quad (4)$$

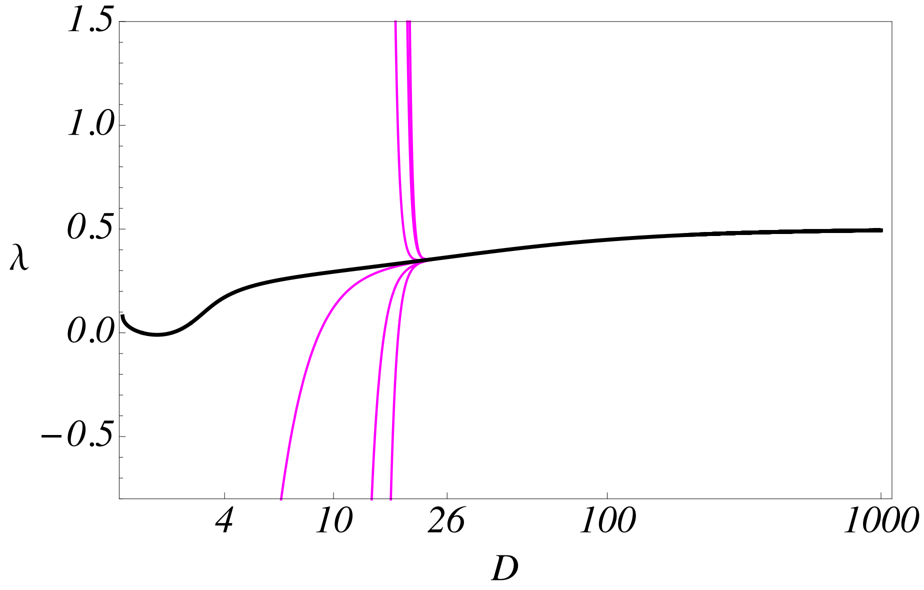
1/D Expansion Results:

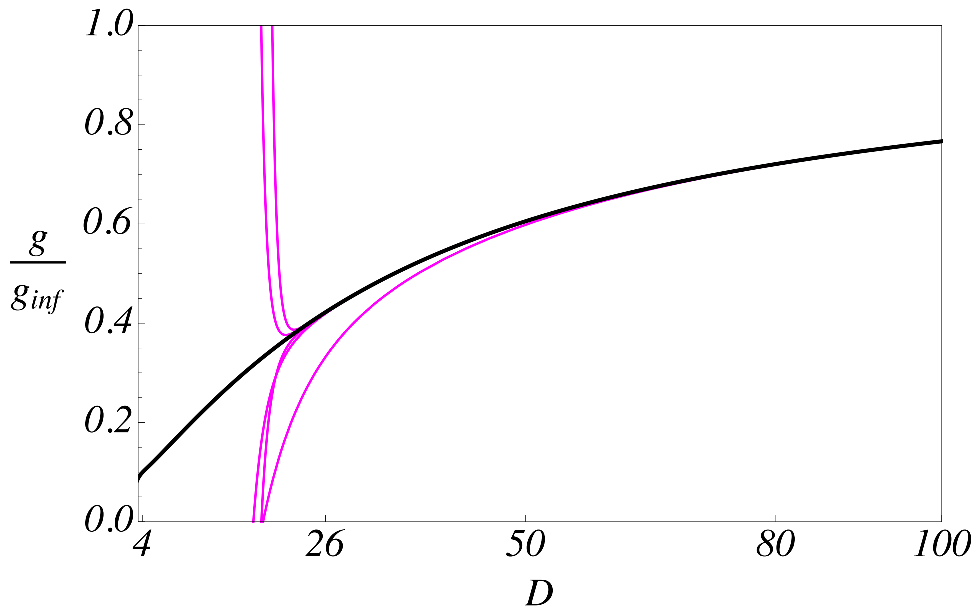
For $0 \leq \alpha < 1$

$$\lambda(D,\alpha) = \frac{1}{2} - \frac{6}{D} + \frac{90}{D^2} - \frac{546-690\alpha}{(1-\alpha)D^3} - \frac{18(399-374\alpha+167\alpha^2)}{(1-\alpha)^2 D^4} + \cdots$$

$$g(D,\alpha) = \frac{6c_D}{D^3}\left(1 - \frac{26}{D} + \frac{425-473\alpha}{(1-\alpha)D^2} - \frac{5422\alpha^2-9500\alpha+3214}{(1-\alpha)^2 D^3} + \cdots\right)$$

$$c_D \equiv (4\pi)^{\frac{D}{2}-1}\Gamma(\tfrac{D}{2}+2)$$



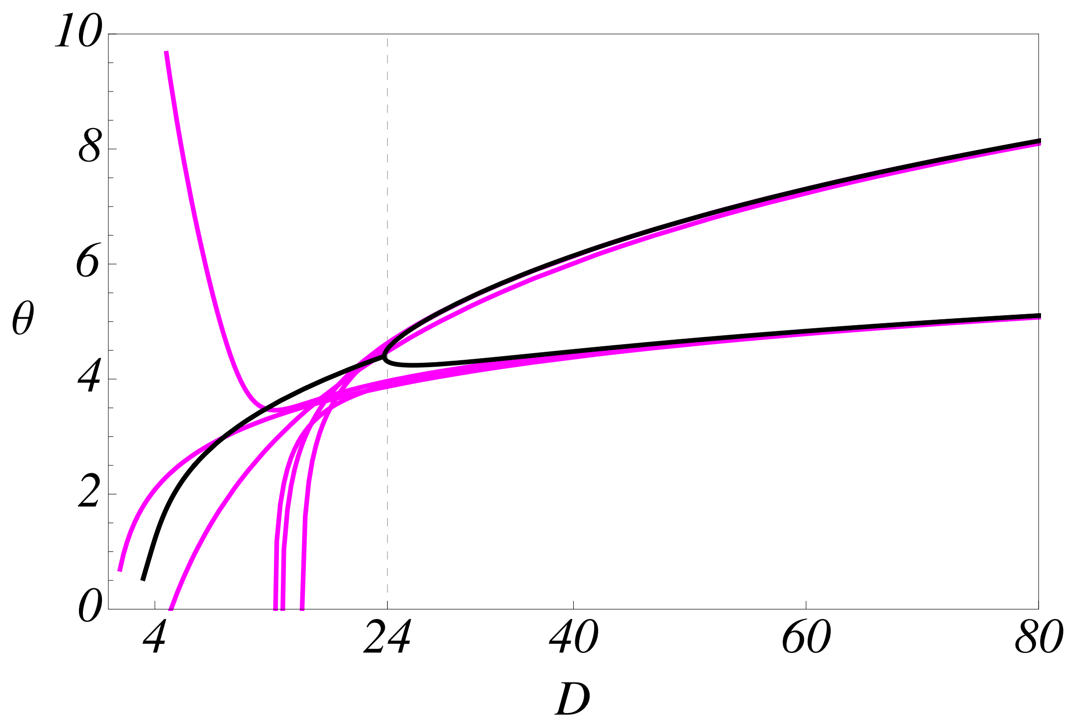


Scaling Exponents

Fixed points are not universal. The eigenvalues of the stability matrix are universal, they define the behaviour of the RG flow in the vicinity of the fixed point. Stability matrix is defined as, $\beta_{i,j} = \frac{\partial \beta_i}{\partial g_j}$

$$\theta_1(D, \alpha < 1) = \frac{D^3}{156}\left(1 + \frac{1}{13D} + \frac{8(6193\alpha - 7207)}{169(\alpha - 1)D^2} + \cdots\right)$$

$$\theta_2(D, \alpha < 1) = 2D\left(1 + \frac{1}{D} + \frac{98}{D^2} - \frac{(920 - 1016\alpha)}{2(\alpha - 1)D^3} + \cdots\right)$$



In a vector dominance gauge i.e. when $\alpha \to \infty$, we find a more stable result,

$$\begin{aligned}
\tilde{\lambda}(D,\infty) &= \frac{1}{2} - \frac{1}{(2D)^{1/2}} + \frac{1}{D} - \frac{5}{(2D)^{3/2}} + \frac{3}{D^2} - \frac{75}{2(2D)^{5/2}} + \frac{13}{D^3} - \cdots \\
\tilde{g}(D,\infty) &= \frac{c_D}{2D^2}\left(1 - \frac{4}{(2D)^{1/2}} + \frac{7}{D} - \frac{44}{(2D)^{3/2}} + \frac{37}{D^2} - \frac{478}{(2D)^{5/2}} + \cdots\right)
\end{aligned}$$

$$\theta_1 = \sqrt{2}D^{3/2}\left(1 - \frac{13}{2D} - \frac{4\sqrt{2}}{D^{3/2}} - \frac{9}{8D^2} - \cdots\right)$$

$$\theta_2 = 2D\left(1 + \frac{2}{D} + \frac{4\sqrt{2}}{D^{3/2}} + \frac{22}{D^2} + \cdots\right)$$

With the “Bimetric” Truncation Approach:

$$\lambda = -\frac{6}{D} - \frac{42}{D^2} + \frac{684}{D^3} + \frac{2256}{D^4} + \cdots$$

$$g = \frac{6c_D}{D^3}\left(1 + \frac{27}{D} + \frac{193}{D^2} - \frac{2321}{D^3} + \frac{3165}{D^4} + \cdots\right)$$

$$\theta_1 = D + \frac{108}{D} - \frac{1392}{D^2} - \frac{29520}{D^3} + \cdots$$

$$\theta_2 = 2D - 8 + \frac{162}{D} - \frac{2824}{D^2} + \frac{29736}{D^3} + \cdots$$

Conclusions

- We reached a successful 1/D expansion of quantum gravity. Fixed points of functional RG exist in a large number of dimensions with a consistent leading order behaviour (gauge independent).

- Scaling exponent ν, due to the gravitational constant always go like1/2D. This is consistent with the lattice result where ν goes to 0 in the large-D, it is also consistent with the previous asymptotic safety result from: *D.Litim arXiv:hep-th/0312114.*

- We can't recover D=4 from this expansion. Series diverge in low dimensions. Reason for that is scaling exponents bifurcate around D=25. The picture changes qualitatively. We should look more deeply into the approximations that we put in our theory.

Back-Up

$$\lambda = \frac{1}{2} - \frac{6}{D} + \frac{90}{D^2} - \frac{678}{D^3} - \frac{2778}{D^4} + \cdots$$

$$g = \frac{6c_D}{D^3}\left(1 - \frac{28}{D} + \frac{525}{D^2} - \frac{6458}{D^3} + \ldots\right)$$

$$g_{\mathrm{NDA}} = (4\pi)^{D/2}\Gamma(D/2)$$

$$\frac{g_{inf}}{g_{\mathrm{NDA}}} = \frac{3(D+2)}{8\pi D^2}$$